



## Mathematical Modeling and Control of Active Suspension System for a Quarter Car Railway Vehicle

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### ABSTRACT

Active suspension is a term where active components are incorporated in a mechanical suspension system. Compared to passive suspension, the active suspension provides improved feature of suspension in terms of ride quality and vehicle handling. While the passive suspension consists of spring and dampers, the active suspension comprises of sensors, actuators and controller with passive system. The actuator in the active suspension provides forces to the suspension which eventually improves ride performance and comfort. This paper presents the early stage of modeling an active suspension of a railway vehicle. The mathematical model of the system is demonstrated and the early suspension control of the vehicle is introduced. As a start only the quarter car vehicle is considered and tested upon random or irregular track input at a speed of 31 m/s. The skyhook control strategy is introduced to the suspension, making it an active system. The performance is then compared to the passive. Preliminary results have shown that the active suspension improves ride quality by reducing the vehicle body accelerations and suspension deflections.

**Keywords:** Active suspension, Skyhook control, Railway vehicle modeling, Non-linear system, Mathematical modeling

## 1. Introduction

In the past vehicle suspension system was a purely mechanical system. The whole system was simple and the ride quality offered was very poor. Improvement of the suspension system by various researches have been done both for automotive and railway vehicle sectors (Sasaki, 1994). Compared to automobile industry active suspension for railway vehicle began much later. Active suspension system for railway had been introduced and analyzed in the 1970's, but has not yet made its convincing breakthrough in operational use (Orvnas, 2008). The reason behind is it's highly expensive and lack of technology advancement. However to stay in competitions with other means of transportation, research for railway industry have concentrated on producing better, high speed, light weight railway vehicles (Goodall and Kortum, 2002). However high speed rail usually create extra forces and accelerations on the vehicle body which place effects on the ride quality. High speed trains have been launched all over the globe. For instance the TGV POS (in French: "Train a Grande Vitess" which means "high-speed train" and POS stand for Paris-Ostfrankreich-Suddeuschland) launched in 2007 by the French national rail company at the highest speed of 357 mph (Foo and Goodall, 2000). The China railway also launched their high speed train in 2010 known as CRH380A which is designed to operate at maximum of 302 mph. The German railway launched their first high speed train in 1991 known as ICE 1 with a speed of 155.3 mph which is then improved to a maximum speed of 226 mph. Meanwhile the Japanese railway has introduced MAGLEV series train which is operated by highly powered magnet. In 2003, JR-Maglev MLX01 did a record as world's highest speed train with a top speed of 561 mph.

A basic suspension system consists of springs, dampers and linkages that connect vehicle to its wheels. The function of suspension systems contribute to the ride, handling and braking devices to increase the safety and driving pleasure, and also to keep the vehicle occupants comfortable and well isolate from road noise, bumps, and vibrations. Practically there are three types of suspension system are mostly used which are the passive, semi-active and active suspension system. The passive suspension system consists of mainly damper and spring. The damper is used to dissipate energy while the spring is used to store the energy. As for the passive system, all of the parameters are fixed. This has affected on the vehicle ride comfort and also vehicle handling meanwhile the semi-active suspension system is similar to passive suspension system. The difference is it has adjustable damping parameter.

Active suspension system on other hand works with the advancement of technology where additional component such as sensors, actuators, and con-

trollers combined with damper and spring which can improve the overall system if it is applied accordingly (Chen et al., 2012);(Goodall, 1997);(Sasaki, 1994).

Active suspension system has many advantages over passive suspension system. In railway technology active suspension system is usually applied to achieve following the goals: (1) increase passenger ride comfort, (2) continue good ride quality although vehicle speed is increased, (3) continue good ride comfort if track conditions are worse, (4) improve curve negotiation to reduce wheel and rail wear, and (5) provide security at higher vehicle speed (Orvnaš, 2011). However in this paper only items (1)-(3) are taken into consideration.

In this preliminary study we have developed active control based on skyhook damping strategy. The actuator assumed in this study is ideal which can provide any amount of force according to track input with infinite bandwidth. But in real case it shows degradation from ideal case at high frequency due to the dynamics nature of itself and finite bandwidth (Foo and Goodall, 2000); (Md Yusof, 010a).

## 2. Modeling of a Quarter Railway vehicle

The dynamics of a real railway vehicle have significant nonlinearities which include the air spring behavior, damper blow offs, and bump stop contact. The quarter car model of railway vehicle is illustrated in Figure 1. The model is consists of a bogie and a set of wheels connected by primary suspension. The air spring has been placed between vehicle body and bogie which includes of an air bag connected to a surge reservoir through a controlling orifice (Oda, 1970). When active suspension system is applied, active forces are being produced by the actuators.

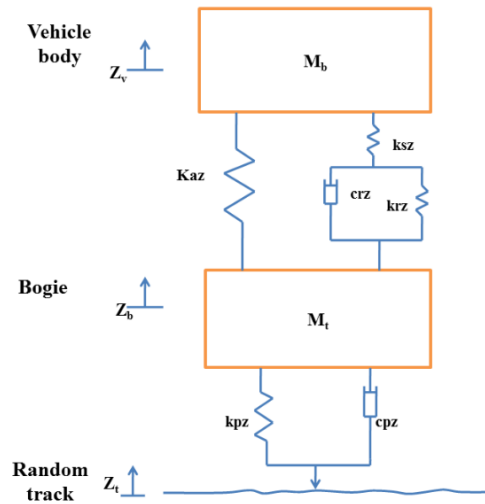


Figure 1: Unicycle passive suspension model

In order to study the vehicle vibration characteristic, the equations of motion based on Newton's second law for each mass are formulated. The vehicle body is influenced by passive secondary suspension components (springs and dampers) and by the active force, if active suspension is implemented; these forces being produced by the actuator. The primary suspensions are influenced by both passive and active forces along with the track inputs. Therefore to model this system, the forces acting on each mass can be used to create the motion equation of each mass.

The air spring model which has been used in this study is presented in Figure 2. The used air spring model is modified by Oda and Nishimura in 1970, which has traditional air spring dynamics with an auxiliary reservoir (Oda, 1970).

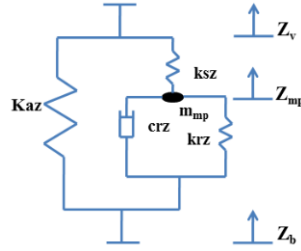


Figure 2: Air spring model

The change of area stiffness ( $k_a$ ) and the air spring mid-point mass ( $m_{mp}$ ) has assumed to be zero because of their small value with respect to the total entire system. The general equation for air spring model can be written as,

$$c_{rz}(\dot{z}_b - \dot{z}_{mp}) + k_{rz}(z_b - z_{mp}) - k_{sz}(z_{mp} - z_v) = 0.$$

Now after re-arranging the above equation it can be written as

$$\dot{z}_{mp} = \frac{k_{rz}}{c_{rz}} z_v - \frac{(k_{sz} - k_{rz})}{c_{rz}} z_{mp} + \frac{k_{rz}}{c_{rz}} z_b + \dot{z}_b. \quad (1)$$

Applying Newton's second law the equations of motion for the system are formulated. The equation of motion for the bounce of vehicle body (balancing force in z direction) can be represented,

$$\ddot{z}_v m_b = k_{sz}(z_{mp} - z_v) + k_{az}(z_b - z_v).$$

After re-arranging the above equation it can be written as

$$\ddot{z}_v = \frac{1}{m_b} (-(k_{az} + k_{sz})z_v + k_{sz}z_{mp} + k_{az}z_b), \quad (2)$$

and the equation of bogie as:

$$\ddot{z}_b m_t = c_{pz}(\dot{z}_t - \dot{z}_b) + k_{pz}(z_t - z_b) - k_{sz}(z_{mp} - z_v) - k_{az}(z_b - z_v),$$

or,

$$\ddot{z}_b = \frac{1}{m_t}((k_{az} + k_{sz})z_v - k_{sz}z_{mp} - (k_{az} + k_{pz})z_b - c_{pz}\dot{z}_b + k_{pz}z_t + c_{pz}\dot{z}_t). \quad (3)$$

"Eq. (1)" to "Eq. (2)" describes the system shown in Figure 1, and can be arranged in a state-space form for system analysis and further controller design

$$\dot{x} = Ax + Bu + G\zeta, \quad (4)$$

$$y = cx + Du + G\zeta, \quad (5)$$

where the state vector,  $x$ , is

$$x = [z_v \quad \dot{z}_v \quad z_{mp} \quad z_b \quad \dot{z}_b \quad z_t], \quad (6)$$

$$\zeta = [\dot{z}_t]^T. \quad (7)$$

The active force input is  $u_{act}$  for the suspension and  $\zeta$  contains the track inputs from the wheel to bogie. The outputs of interest are the car body accelerations to determine the ride quality and the secondary suspension deflections, given by

$$y = [z_v \quad \dot{z}_v \quad \ddot{z}_v \quad \dot{z}_b \quad z_b \quad f_{act}]^T, \quad (8)$$

The nominal values of the vehicle parameters and the state-space model for this vehicle are extracted from a typical railway vehicle British Rail MK III coach is shown in the appendix (Table 4) (Md Yusof, 010b). And the variable used in this unicycle model is tabulated in Table 1.

To verify the vehicle model a step response analysis is performed. Figure 3 shows the step track input. Where input track velocity is given a pulse height of .2 m/s for 0.3 s that means the track position increase about 6 cm. The

Table 1: List of variable used in the unicycle model.

Variable	Definitions
$z_v$	Vertical displacement for vehicle body
$z_b$	Vertical displacement for bogie
$z_{mp}$	Vertical displacement for air spring mid-point mass
$z_t$	Random track input
$k_{pz}$	Primary spring stiffness
$c_{pz}$	Primary damping
$k_{rz}$	Secondary reservoir stiffness
$c_{rz}$	Secondary damping
$k_{sz}$	Air spring volume stiffness
$k_{az}$	Air spring change of area stiffness
$m_b$	Mass of the vehicle body
$m_t$	Mass of the bogie
$m_{mp}$	Air spring mid-point mass

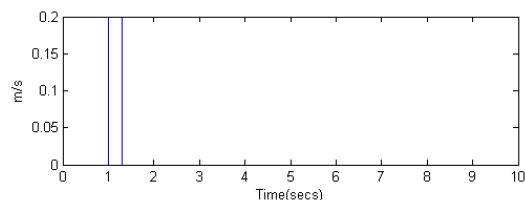


Figure 3: The step input

gradient of 1% is considered for typical railway track. The speed of the vehicle is assumed as  $31m/s$  with acceleration limit of  $0.2m/s^2$ .

Figure 4 represents the acceleration response of the vehicle body where the steady state acceleration falls into zero and Figure 5 represents the velocity of the vehicle body and bogie. The steady state displacement of the vehicle body and bogie increases to 6 cm which means the vehicle body and bogie follows the track position. The phenomenon is illustrated in Figure 6.

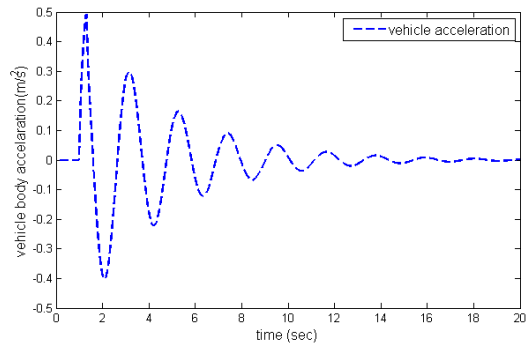


Figure 4: The vehicle body acceleration for step input

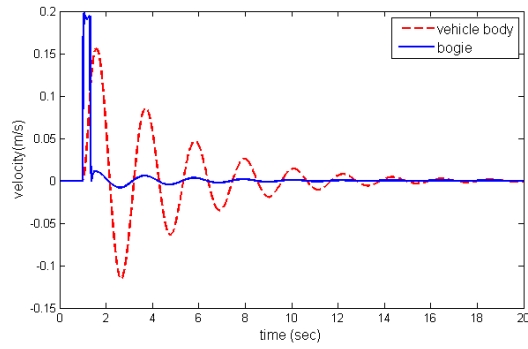


Figure 5: The vehicle body and bogie velocity response of a step input



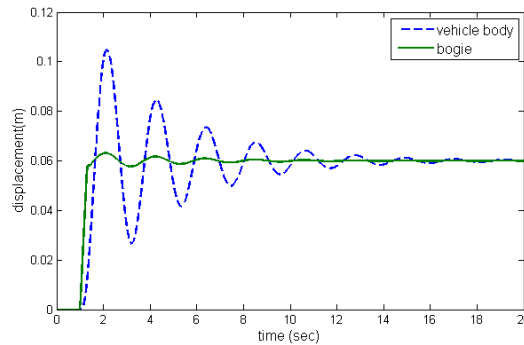


Figure 6: The vehicle body and bogie displacement for step track input

The performance of the vehicle model has been assessed for random track inputs which represents the misalignment of the rail track. The random track input is accepted by the UK Railway and also by other railway administrations. The recording is done based on a number of routes using high speed recording coaches. The track roughness factor  $A_r$  is assumed to be  $2.5 \times 10^7$ . The root mean square (r.m.s) values of the vertical acceleration of the vehicle body are used to evaluate the ride quality. All the accelerations are expressed in  $\%g$ , the forces in  $kN$  and the suspension deflections in mm responding to the stochastic inputs. The ride quality of the passive vehicle model is tabulated in Table 2.

Table 2: R.M.S results of passive vehicle

	Accelerations ( $\%g$ )	Suspension Deflections (mm)	Force(kN)
Passive	2.284	55.2	—

### 3. Active secondary suspension control strategy

The concept of skyhook is an imaginary damper connected to an inertial reference in the sky. The main strategy is active force is proportional to the relative vertical velocity across the suspension (Li, 1999); (Chen et al., 2012). Figure 7 illustrates the concept of skyhook damping.

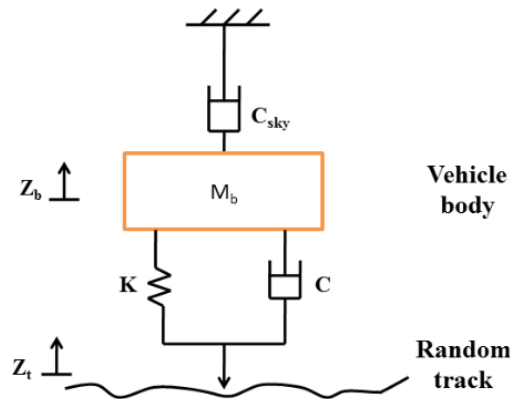


Figure 7: Ideal skyhook damping

So control action of the skyhook damper is dependent on the absolute velocity of the vehicle body  $z_b$  :

$$f_{act} = -c_{sky}\dot{z}_b,$$

Where  $f_{act}$ , is the control force and  $c_{sky}$  is the skyhook damping gain. But practically the measurement of absolute velocity is not possible. An accelerometer is used to measure the acceleration of vehicle body. To evaluate absolute velocity from accelerometer an integrator with a second order Butterworth high pass filter has been chosen. The high pass filter is used to form a self-zeroing integrator, which help to minimize the suspension deflection in deterministic input. For achieving good ride quality a cut off frequency of .1 Hz and a damping ratio of 70% is applied. Figure 8 illustrates the practical implementation of skyhook damping.

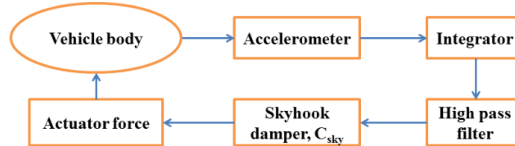


Figure 8: Practical skyhook implantation

In this study the local skyhook control strategy is included for the quarter

car model. Figure 9 shows the schematic diagram of local skyhook control strategy.

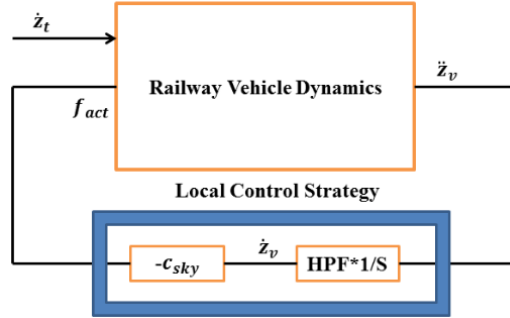


Figure 9: Local skyhook control strategy

According to this strategy the acceleration ( $\ddot{z}_v$ ) of the vehicle body is measured by accelerometer. Then the accelerometer signal passes through integrator with high pass filter to obtain desired absolute velocity. The actuator demand ( $f_{act}$ ) is produced by amplified the absolute velocity with skyhook damping gain ( $c_{sky}$ ). After including skyhook damping to the passive system of the quarter car model the equation of the vehicle body becomes as

$$\ddot{z}_v = \frac{1}{m_b} (-(k_{az} + k_{sz})z_v + k_{sz}k_{mp} + k_{az}z_b - c_{sky}\dot{z}_v), \quad (9)$$

and the equation of the bogie becomes

$$\ddot{z}_b = \frac{1}{m_t} ((k_{az} + k_{sz})z_v - k_{sz}k_{mp} - (k_{az} + k_{pz})z_b - c_{pz}\dot{z}_b + k_{pz}z_t + c_{pz}\dot{z}_t + c_{sky}\dot{z}_v). \quad (10)$$

Figure 10 represents the quarter car model of railway with ideal actuator.

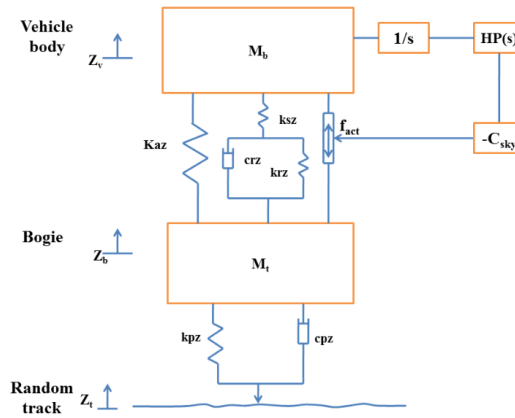


Figure 10: Unicycle active suspension model

Figure 11 represents the trade-off between acceleration (%g) and suspension deflection (mm), and the points indicating the results for the passive and active suspensions with different skyhook damping value where  $(c_{sky}) = 30\text{ kN/m.s}^{-1}$  gives the best outcome compare to other  $(c_{sky})$  values. The r.m.s results are tabulated in Table 3.

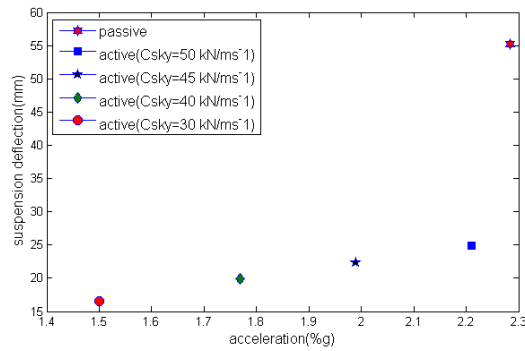


Figure 11: Trade-off between passive and active suspension

Table 3: Active suspension results

	Accelerations ( $\%g$ )	Suspension Deflections(mm)	Force(kN)
Active( $c_s k_y=50kN/ms^{-1}$ )	2.211	24.9	3.37
Active( $c_s k_y=45kN/ms^{-1}$ )	1.990	22.4	3.03
Active( $c_s k_y=40kN/ms^{-1}$ )	1.769	19.9	2.69
Active( $c_s k_y=30kN/ms^{-1}$ )	1.501	16.6	2.08

Table 4: Vehicle Parameters.

Parameters	Values
Body mass, $m_b$	38000 kg
Air spring mass, $m_{mp}$	5 kg
Bogie mass, $m_t$	2500 kg
Air spring change of area stiffness, $k_{az}$	$0 Nm^{-1}$
Air spring volume stiffness, $k_{sz}$	$1160000 Nm^{-1}$
Secondary reservoir stiffness, $k_{rz}$	$508000 Nm^{-1}$
Secondary damping, $c_{rz}$	$50000 Nsm^{-1}$
Primary spring stiffness, $k_{pz}$	$4935000 Nm^{-1}$
Primary damping, $c_{pz}$	$50740 Nsm^{-1}$

Figure 12 shows the vehicle body accelerations for active and passive suspension system with respect to time for random track inputs. And Figure 13 represents the vehicle body displacement for random track inputs.

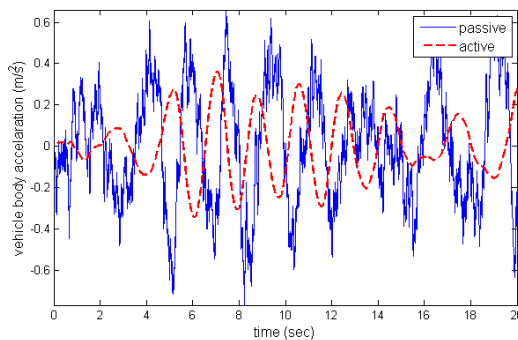


Figure 12: The vehicle body acceleration for random track input

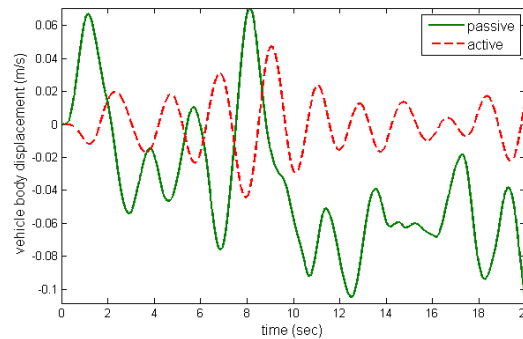


Figure 13: The vehicle body displacement for random track input

## Conclusion

This paper has concentrated on introducing active control for vertical secondary suspension of a railway vehicle. Using the skyhook control law the performance of railway vehicle suspension has improved compared to the passive system. The mathematical modeling of the vehicle that has been developed was based on simplified quarter car. In this paper only concentrates on the bounce mode to evaluate the performance of the active secondary suspension system as the pitch mode will only introduced in the full vehicle model which will be looked in later. The results are presented by overall r.m.s values of the vertical vehicle body accelerations. From the result it is clear that active suspension system has the ability to improve the ride quality of railway vehicle by reducing the vehicle body accelerations and suspension deflections. However to implement active suspension, real actuators needs to be taken into consideration. In real case actuators it-self have its own dynamic nature which will make the ride quality deteriorate from the ideal. As a result the performance of the active suspension system also decreases. This important aspect of including real actuator dynamics into the suspension will be investigated in the next part of our research.

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